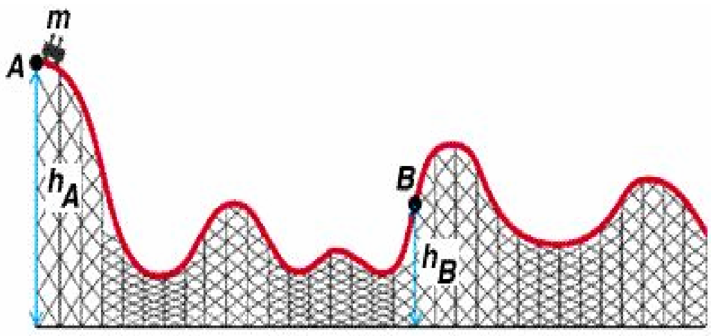
Physics 151 Class Exercise: Energy-KEY



1. (a) A 236 kg roller coaster car is released from rest from position A where h_A = 40 m. What is the velocity of the car when it gets to position B where h_B = 18 m? You should work this part of the problem with the ZLP at ground. There is no friction in this problem.

$E_A = E_B$
$mgh_A = mgh_B + KE_B$

$$mgh_A = mgh_B + \frac{1}{2}mv_B^2$$

$$gh_A = gh_B + \frac{1}{2}v_B^2$$

$$v_B = \sqrt{2g(h_A - h_B)}$$

$$= \sqrt{2\left(9.81\frac{m}{s^2}\right)(40m - 18m)} = 20.8\frac{m}{s}$$

(b) Rework part (a) with the ZLP at the level of position B.

$E_A = E_B$
$mgh_{A\ \text{above}\ B} = KE_B$

$$mgh_A = mgh_B + \frac{1}{2}mv_B^2$$

$$2gh_{AaboveB} = v_B^2$$

$$v_B = \sqrt{2gh_{AaboveB}}$$

$$v_B = \sqrt{2\left(9.81\frac{m}{s^2}\right)(22m)} = 20.8\frac{m}{s}$$

(c) Imagine instead that the car rounded position A with a velocity of 15 m/s. What would be the velocity in this instance when the car reaches position B?

$E_A = E_B$
$KE_A + mgh_A = mgh_B + KE_B$

$$\frac{1}{2}mv_B^2 + mgh_A = mgh_B + \frac{1}{2}mv_B^2$$

Where I have assumed that the ZLP is back on the ground.

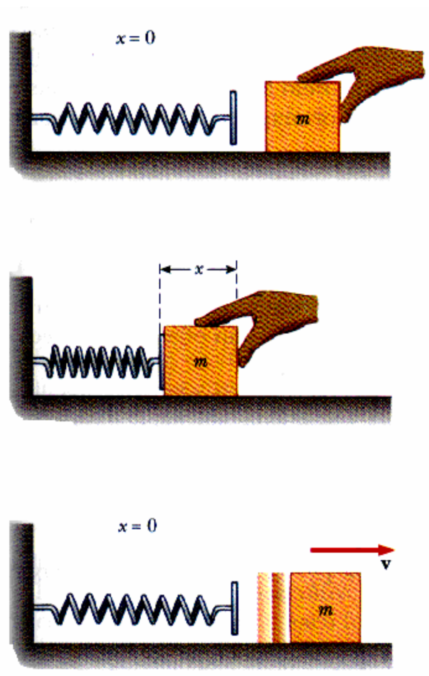
$$\frac{1}{2}v_A^2 + gh_A = gh_B + \frac{1}{2}v_B^2$$

$$v_A^2 + 2gh_A = 2gh_B + v_B^2$$

$$v_B = \sqrt{v_A^2 + 2g(h_A - h_B)}$$

$$= \sqrt{\left(15\frac{m}{s}\right)^2 + 2\left(9.81\frac{m}{s^2}\right)(40m - 18m)} = 25.6\frac{m}{s}$$

2. A 12 N/m horizontal spring on a frictionless surface is shown in the first panel of the illustration in its equilibrium position. A 1.2 kg mass is pushed against the spring compressing it a distance of x = 8.2 cm in the second panel.



(a) The hand is then quickly removed? What will be the velocity of the mass when it reaches the equilibrium position?

$$E_{amplitude} = E_{equilibrium}$$
$$KE_{amplitude} + PE_{amplitude} = KE_{equilibrium} + PE_{equilibrium}$$
$$PE_{amplitude} = KE_{equilibrium}$$
$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$$
$$v = x_0\sqrt{\frac{k}{m}} = (0.082m)\sqrt{\frac{12\frac{N}{m}}{1.2\,kg}} = 0.26\frac{m}{s}$$

(b) What velocity did the mass have at x = 6.0 cm along the way?

At x = 6.0 cm the mass will have a combination of potential and kinetic energy.

$$E_{amplitude} = E_{6.0cm}$$
$$PE_{amplitude} = KE_{6.0cm} + PE_{6.0cm}$$
$$\frac{1}{2}kx_0^2 = \frac{1}{2}kx_{6.0cm}^2 + \frac{1}{2}mv^2$$
$$kx_0^2 = kx_{6.0cm}^2 + mv^2$$
$$v = \sqrt{\left(x_0^2 - x_{6.0cm}^2\right)\frac{k}{m}} = \sqrt{\left[(0.082m)^2 - (0.06m)^2\right]\frac{12\frac{N}{m}}{1.2\,kg}} = 0.18\frac{m}{s}$$